INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 42, Northern Spring 2021 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Is it possible that a product of 9 consecutive positive integers is equal to a sum of 9 consecutive (not necessarily the same) positive integers? (4 points)

2. Let AX and BZ be altitudes of the triangle ABC. Let AY and BT be its angle bisectors. It is given that angles XAY and ZBT are equal. Does this necessarily imply that ABC is isosceles? (4 points)

3. Maria has a balance scale that can indicate which of its pans is heavier or whether they have equal weight. She also has 4 weights that look the same but have masses of 1001, 1002, 1004 and 1005 grams. Can Maria determine the mass of each weight in 4 weighings? The weights for a new weighing may be picked when the results of the previous ones are known. (4 points)

4. (a) Is it possible to split a square into 4 isosceles triangles such that no two of them are congruent? (3 points)

   (b) Is it possible to split an equilateral triangle into 4 isosceles triangles such that no two of them are congruent? (3 points)

5. A rectangular board consists of square cells. Some of the cells are occupied by dominoes such that each domino occupies two adjacent cells and none of the dominoes are touching, even at a vertex. The bottom left and top right cells of the board are vacant. A token starts at the bottom left cell and can move to a cell adjacent by side: one step to the right or upwards at each turn. Is it always possible to move from the bottom left to the top right cell without passing through dominoes if the size of the board is

   (a) 100×101 cells; (2 points)

   (b) 100×100 cells? (4 points)